排灌机械工程学报
Journal of Drainage and Irrigation Machinery Engineering
2012 年 9 月 Sept. 2012 第 30 卷 第 5 期 Vol. 30 No. 5

doi:10.3969/j. issn. 1674 -8530. 2012. 05. 021

任意曲线坐标系下二维浅水方程的数值模拟

吴卫国, 薛世峰
(中国石油大学(华东)工程力学系, 山东 青岛 266555)

摘要: 采用拟合边界曲线的方法来构造天然河道、湖泊、港口河口以及海湾的复杂边界问题,建立任意正交曲线网格以克服由于复杂边界而引起的计算困难. 在此基础上,推导出任意曲线坐标系下的二维浅水方程、湍流动能方程和湍流动能耗散方程;应用有限差分方法对该方程组进行数值离散,并用交错方向隐式格式实现在计算区域内对任意曲线坐标系下的二维浅水方程进行数值求解. 为了验证在任意曲线坐标系下二维浅水方程数值求解方法的可靠性、正确性,以 De Vriend 的 180°平面弯道水槽试验物理模型为例进行数值模拟,结果表明,数值计算的结果和 De Vriend 的试验结果相当吻合,最大绝对误差值约为 10^{-2},因此,数值计算方法合理可行,可为任意复杂边界的天然河道、湖泊等水域的水动力研究提供有效的计算方法.

关键词: 浅水方程;曲线坐标;交错方向隐式格式;复杂边界;数值模拟

中图分类号: S277.9; TV145.21 **文献标志码:** A **文章编号:** 1674 -8530(2012)05 -0598 -05

Numerical simulation of 2D shallow water equation in arbitrary curvilinear coordinates

Wu Weiguo, *Xue Shifeng*
(Department of Engineering Mechanics, China University of Petroleum, Qingdao, Shandong 266555, China)

Abstract: The problems with complex boundary shapes, such as natural river channels, lakes, estuaries and bays, were solved by using the boundary curve fitting methods. Arbitrary orthogonal curvilinear grid was established to overcome the computational difficulties caused by those complex boundaries. Then, a set of derived equations in the arbitrary curvilinear coordinates, including 2D shallow water equation, turbulence kinetic energy equation and dissipation rate equation etc. were numerically discreted by the finite difference method. In addition, the 2D shallow water equation was numerically solved within the computational domain by using the alternating direction implicit (ADI) difference scheme. In order to verify the reliability and correctness of the method, the De Vriend's 180° plane curve flume experiment model was adopted as an example to implement the numerical simulations. Finally, the simulation outcomes are in excellent agreement with that experimental results with a maximum error as large as approximate 10^{-2}, indicating that the numerical method in this paper is reasonable and feasible. Hence, the method will provide an efficient way for calculating hydrodynamics of water bodies with arbitrary complex boundaries, such as natural river channels and lakes.

Key words: shallow water equation; curvilinear coordinates; ADI difference scheme; complex boundarry; numerical simulation

---

收稿日期: 2012 -04 -01
基金项目: 国家自然科学基金资助项目(11172143)
作者简介: 吴卫国(1967—),男,江苏常熟人,博士研究生(wwg1967g@163.com),主要从事环境力学研究.
薛世峰(1963—),男,河北新河人,教授,博士生导师(xuesf@126.com),主要从事工程力学研究.

随着我国经济的快速发展、社会进步以及城市化进程的加快,城市污水及工业废水排放量也在加大,且排放种类也具多样性,它们是导致我国水环境质量恶化的根本原因. 弄清楚河道、港口河口、湖泊等水动力因素,是解决水环境问题的重要基础. 天然河道、海湾其边界曲折、地形复杂,采用任意曲线坐标是解决问题的根本途径之一. 一般地,天然河道中的水流是三维的,但天然河道的深度与宽度相比很小,可将三维水流运动简化为平面浅水运动处理. 因此,研究任意曲线坐标系下二维浅水方程的数值计算很有意义.

对二维非定常流进行研究,国内外许多专家学者在这方面做了大量的工作. Thompson 等[1]提出贴体坐标法,使其计算区域边界与真实边界密切贴合,并在工程实践中得到广泛的应用. 其后,Stelling[2]对平面浅水方程详细地做了研究,得到一些有意义的结果. 王船海等[3]提出了天然河道非恒定流场的通用数学模型,合理解决了动边界的跟踪问题. De Vriend[4]在 180°弯道水槽中试验,获得了较丰富的资料. 王如云等[5]针对涌波现象导出二维曲线坐标系下的守恒型方程,解决了急流过窄河道产生涌波结构问题. 黄炳彬等[6]在直角坐标系下发展了一套处理复杂边界的斜对角笛卡儿方法. 吴修广等[7]采用 Laplace 方程坐标变换方法研究了浅水流动问题. Shi 等[8]在曲线坐标系下研究了表面波的传播机理问题,获得较好的结果. 在过去的研究成果中,虽然获得了一些有意义的结果,但都存在一个共同的局限,普适性较差.

文中将任意曲线坐标系下的二维浅水方程和 $k-\varepsilon$方程联立求解,目的是为了真实模拟任意复杂天然河道、湖泊等地形多变的问题,提高计算精度,加大计算时间步长,增强计算的稳定性能,为任意复杂边界的天然河道水动力计算提供合理可行的方法.

1　基本理论

1.1　浅水方程

在曲线坐标系下二维浅水控制方程:水位方程、动量方程、湍流动能和湍流动能耗散方程(简称 $k-\varepsilon$ 方程)分别如下:

$$\frac{\partial h}{\partial t}+\frac{1}{H_1H_2}\left(\frac{\partial H_2hu}{\partial \xi}+\frac{\partial H_1hv}{\partial \eta}\right)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{u}{H_1}\frac{\partial u}{\partial \xi}+\frac{v}{H_2}\frac{\partial u}{\partial \eta}+\frac{v}{H_1H_2}\left(u\frac{\partial H_1}{\partial \eta}-v\frac{\partial H_2}{\partial \xi}\right)+\frac{g}{H_1}\frac{\partial (h+z)}{\partial \xi}+gm^2u\frac{\sqrt{u^2+v^2}}{h^{4/3}}=(\nu+\nu_t)\left(\frac{1}{H_1}\frac{\partial D}{\partial \xi}-\frac{1}{H_2}\frac{\partial \zeta}{\partial \eta}\right), \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{u}{H_1}\frac{\partial v}{\partial \xi}+\frac{v}{H_2}\frac{\partial v}{\partial \eta}+\frac{u}{H_1H_2}\left(v\frac{\partial H_2}{\partial \xi}-u\frac{\partial H_1}{\partial \eta}\right)+\frac{g}{H_2}\frac{\partial (h+z)}{\partial \eta}+gm^2v\frac{\sqrt{u^2+v^2}}{h^{4/3}}=(\nu+\nu_t)\left(\frac{1}{H_2}\frac{\partial D}{\partial \eta}+\frac{1}{H_1}\frac{\partial \zeta}{\partial \xi}\right), \tag{3}$$

$$\frac{\partial k}{\partial t}+\frac{u}{H_1}\frac{\partial k}{\partial \xi}+\frac{v}{H_2}\frac{\partial k}{\partial \eta}=\frac{1}{H_1}\frac{\partial}{\partial \xi}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{1}{H_1}\frac{\partial k}{\partial \xi}\right]+\frac{1}{H_2}\frac{\partial}{\partial \eta}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{1}{H_2}\frac{\partial k}{\partial \eta}\right]+P_k-\varepsilon, \tag{4}$$

$$\frac{\partial \varepsilon}{\partial t}+\frac{u}{H_1}\frac{\partial \varepsilon}{\partial \xi}+\frac{v}{H_2}\frac{\partial \varepsilon}{\partial \eta}=\frac{1}{H_1}\frac{\partial}{\partial \xi}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{1}{H_1}\frac{\partial \varepsilon}{\partial \xi}\right]+\frac{1}{H_2}\frac{\partial}{\partial \eta}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{1}{H_2}\frac{\partial \varepsilon}{\partial \eta}\right]+\frac{\varepsilon}{k}(c_1P_k-c_2\varepsilon), \tag{5}$$

其中:

$$H_1=\sqrt{\left(\frac{\partial x}{\partial \xi}\right)^2+\left(\frac{\partial y}{\partial \xi}\right)^2},$$

$$H_2=\sqrt{\left(\frac{\partial x}{\partial \eta}\right)^2+\left(\frac{\partial y}{\partial \eta}\right)^2},$$

$$D=\nabla\cdot\boldsymbol{v}=\frac{1}{H_1H_2}\left(\frac{\partial uH_2}{\partial \xi}+\frac{\partial vH_1}{\partial \eta}\right),$$

$$\zeta=(\nabla\times\boldsymbol{v})_3=\frac{1}{H_1H_2}\left(\frac{\partial vH_2}{\partial \xi}-\frac{\partial uH_1}{\partial \eta}\right),$$

以上式中:h 为水位;z 为河床高度;g 为重力加速度;u,v 分别为曲线坐标 ξ,η 方向的速度;t 为时间;m 为曼宁系数;ζ 为 z 方向涡度;D 为散度;ν 是运动黏度;ν_t 为湍流运动黏度,$\nu_t=c_\mu k^2/\varepsilon$,$c_\mu=0.09$ 为经验系数,k 为湍动能,ε 为耗散率;P_k 为湍动能产生项;$c_1=1.44$,$c_2=1.92$,$\sigma_k=1.0$,$\sigma_\varepsilon=1.3$ 均为经验系数.

1.2　边界条件

壁面无滑移边界条件 $u=0$,$v=0$,$\frac{\partial h}{\partial n}=0$;入流流量给定,采用 Spalding 剖度与湍流外层律相结合,确定入流速度分布;用二阶精度线性外插求得水位 h;出流水位 h 给定,出流速度 u 与 v 采二阶精度线性外插进行.

$k-\varepsilon$ 方程的边界条件为:近边界点给定

$\varepsilon = \frac{\sqrt{(0.3k)^3}}{0.4\Delta y}$，壁面 $k=0$，$\varepsilon=0$；入流进口：$k=0.003\ 75(u^2+v^2)$，$\varepsilon=\frac{0.09\sqrt{k^3}}{0.05h}$；$k$ 和 ε 出流条件采用二阶精度的线性外插进行.

2 数值方法

2.1 水位方程和动量方程的数值离散

浅水方程是采用二阶精度的交错方向隐式格式(ADI). 首先,用隐式联立求解水位方程和 u 向速度动量方程,然后显式求解 v 向速度动量方程;其次,则联立隐式求解水位方程和 v 向速度动量方程,再显式求解 u 向速度动量方程.

第 1 步离散方程计算:

$$\frac{h_{i,j}^{n+1/2}-h_{i,j}^{n}}{\Delta t/2}+\frac{1}{H_1H_2}\{[(H_2)_{i+1/2,j}h_{i+1/2,j}^{n}u_{i+1/2,j}^{n+1/2}-(H_2)_{i-1/2,j}h_{i-1/2,j}^{n}u_{i-1/2,j}^{u+1/2}]+[(H_1)_{i,j+1/2}h_{i,j+1/2}^{n}v_{i,j+1/2}^{n}-(H_1)_{i,j-1/2}h_{i,j-1/2}^{n}v_{i,j-1/2}^{n}]\}=0, \tag{6}$$

$$\frac{u_{i+1/2,j}^{n+1/2}-u_{i+1/2,j}^{n}}{\Delta t/2}+\frac{u}{H_1}\frac{\partial u}{\partial \xi}+\frac{v}{H_2}\frac{\partial u}{\partial \eta}+\frac{v_{i+1/2,j}^{n}}{H_1H_2}\left(u_{i+1/2,j}^{n}\frac{\partial H_1}{\partial \eta}-v_{i+1/2,j}^{n}\frac{\partial H_2}{\partial \xi}\right)+\frac{g}{H_1}(h_{i+1,j}^{n+1/2}-h_{i,j}^{n+1/2})+\frac{g}{H_1}\frac{\partial z}{\partial \xi}+gm^2u_{i+1/2,j}^{n}\frac{\sqrt{(u_{i+1/2,j}^{n})^2+(v_{i+1/2,j}^{n})^2}}{h^{4/3}}=(\nu+\nu_t)\left(\frac{1}{H_1}\frac{\partial D}{\partial \xi}-\frac{1}{H_2}\frac{\partial \zeta}{\partial \eta}\right)_{i+1/2,j}^{n}, \tag{7}$$

$$\frac{v_{i,j+1/2}^{n+1/2}-v_{i,j+1/2}^{n}}{\Delta t/2}+\frac{u}{H_1}\frac{\partial v}{\partial \xi}+\frac{v}{H_2}\frac{\partial v}{\partial \eta}+\frac{u_{i,j+1/2}^{n+1/2}}{H_1H_2}\left(v_{i,j+1/2}^{n}\frac{\partial H_2}{\partial \xi}-u_{i,j+1/2}^{n+1/2}\frac{\partial H_1}{\partial \eta}\right)+\frac{g}{H_2}(h_{i,j+1}^{n+1/2}-h_{i,j}^{n+1/2})+\frac{g}{H_2}\frac{\partial z}{\partial \eta}+gm^2v_{i,j+1/2}^{n}\frac{\sqrt{(u_{i,j+1/2}^{n+1/2})^2+(v_{i,j+1/2}^{n})^2}}{h^{4/3}}=(\nu+\nu_t)\left(\frac{1}{H_2}\frac{\partial D}{\partial \eta}+\frac{1}{H_1}\frac{\partial \zeta}{\partial \xi}\right)_{i,j+1/2}^{n}, \tag{8}$$

以上式中:非线性项用显式迎风格式计算,水位方程用隐式,在(i,j)格点上;u 方程采用隐式,且在$(i+1/2,j)$格点上;联立求解 $h^{n+1/2}$ 和 $u^{n+1/2}$;v 方程采用显式,在$(i,j+1/2)$格点上,通过求解 $v^{n+1/2}$,完成第 1 步计算.

第 2 步离散方程计算:

$$\frac{h_{i,j}^{n+1}-h_{i,j}^{n+1/2}}{\Delta t/2}+\frac{1}{H_1H_2}\{[(H_2)_{i+1/2,j}h_{i+1/2,j}^{n+1/2}u_{i+1/2,j}^{n+1/2}-(H_2)_{i-1/2,j}h_{i-1/2,j}^{n+1/2}u_{i-1/2,j}^{n+1/2}]+[(H_1)_{i,j+1/2}h_{i,j+1/2}^{n+1/2}v_{i,j+1/2}^{n+1}-(H_1)_{i,j-1/2}h_{i,j-1/2}^{n+1/2}v_{i,j-1/2}^{n+1}]\}=0, \tag{9}$$

$$\frac{v_{i,j+1/2}^{n+1}-v_{i,j+1/2}^{n+1/2}}{\Delta t/2}+\frac{u}{H_1}\frac{\partial v}{\partial \xi}+\frac{v}{H_2}\frac{\partial v}{\partial \eta}+\frac{u_{i,j+1/2}^{n+1/2}}{H_1H_2}\left(v_{i,j+1/2}^{n+1/2}\frac{\partial H_2}{\partial \xi}-u_{i,j+1/2}^{n+1/2}\frac{\partial H_1}{\partial \eta}\right)+\frac{g}{H_2}(h_{i,j+1}^{n+1}-h_{i,j}^{n+1})+\frac{g}{H_2}\frac{\partial z}{\partial \eta}+gm^2v_{i,j+1/2}^{n+1/2}\frac{\sqrt{(u_{i,j+1/2}^{n+1/2})^2+(v_{i,j+1/2}^{n+1/2})^2}}{h^{4/3}}=(\nu+\nu_t)\left(\frac{1}{H_2}\frac{\partial D}{\partial \eta}+\frac{1}{H_1}\frac{\partial \zeta}{\partial \xi}\right)_{i,j+1/2}^{n+1/2}, \tag{10}$$

$$\frac{u_{i+1/2,j}^{n+1}-u_{i+1/2,j}^{n+1/2}}{\Delta t/2}+\frac{u}{H_1}\frac{\partial u}{\partial \xi}+\frac{v}{H_2}\frac{\partial u}{\partial \eta}+\frac{v_{i+1/2,j}^{n+1}}{H_1H_2}\left(u_{i+1/2,j}^{n+1/2}\frac{\partial H_1}{\partial \eta}-v_{i+1/2,j}^{n+1}\frac{\partial H_2}{\partial \xi}\right)+\frac{g}{H_1}(h_{i+1,j}^{n+1}-h_{i,j}^{n+1})+\frac{g}{H_1}\frac{\partial z}{\partial \xi}+gm^2u_{i+1/2,j}^{n+1/2}\frac{\sqrt{(u_{i+1/2,j}^{n+1/2})^2+(v_{i+1/2,j}^{n+1})^2}}{h^{4/3}}=(\nu+\nu_t)\left(\frac{1}{H_1}\frac{\partial D}{\partial \xi}-\frac{1}{H_2}\frac{\partial \zeta}{\partial \eta}\right)_{i+1/2,j}^{n+1/2}, \tag{11}$$

以上式中:非线性项采用显式迎风格式计算,水位方程用隐式,在(i,j)格点上;v 方程采用隐式,且在$(i,j+1/2)$格点上;联立求解 h^{n+1} 和 v^{n+1};u 方程采用显式,且在$(i+1/2,j)$格点上,通过求解 u^{n+1},完成第 1 步时间步长计算.

2.2 $k-\varepsilon$ 方程的数值离散

下一步利用 $k-\varepsilon$ 方程求解,得 k 和 ε,该方程同样用 ADI 格式,其通式方程为

$$\frac{\partial \varphi}{\partial t}+\frac{u}{H_1}\frac{\partial \varphi}{\partial \xi}+\frac{v}{H_2}\frac{\partial \varphi}{\partial \eta}=\frac{1}{H_1}\frac{\partial}{\partial \xi}\left[\nu z\frac{1}{H_1}\frac{\partial \varphi}{\partial \xi}\right]+\frac{1}{H_2}\frac{\partial}{\partial \eta}\left[\nu z\frac{1}{H_2}\frac{\partial \varphi}{\partial \eta}\right]+R, \tag{12}$$

式中:未知数 $\varphi=k$ 或 $\varphi=\varepsilon$;对 k 方程,$\nu z=\nu+\nu_t/\sigma_k$,$R=P_k-\varepsilon$;对 ε 方程,$\nu z=\nu+\nu_t/\sigma_\varepsilon$,$R=\frac{\varepsilon}{k}(c_1P_k-c_2\varepsilon)$.

首先,对方程(12)在 ξ 向用隐式格式离散,且

对流项采用迎风格式，具体步骤如下：

$u>0$，令

$$RHS2=-\frac{\nu z_{i+1/2,j}}{(H_1)^2(\Delta\xi)^2}\varphi_{i+1,j}^{n+1/2}+\left[\frac{\Delta t}{2}+\frac{2\nu z_{i,j}}{(H_1)^2(\Delta\xi)^2}+\frac{u_{i,j}}{(H_1)(\Delta\xi)}\right]\varphi_{i,j}^{n+1/2}-\left[\frac{\nu z_{i-1/2,j}}{(H_1)^2(\Delta\xi)^2}+\frac{u_{i,j}}{(H_1)(\Delta\xi)}\right]\varphi_{i-1,j}^{n+1/2};\tag{13}$$

$u<0$，令

$$RHS2=-\frac{\nu z_{i-1/2,j}}{(H_1)^2(\Delta\xi)^2}\varphi_{i-1,j}^{n+1/2}\times\left[-\frac{\nu z_{i+1/2,j}}{(H_1)^2(\Delta\xi)^2}+\frac{u_{i,j}}{(H_1)(\Delta\xi)}\right]\varphi_{i+1,j}^{n+1/2}+\left[\frac{\Delta t}{2}-\frac{2\nu z_{i,j}}{(H_1)^2(\Delta\xi)^2}-\frac{u_{i,j}}{(H_1)(\Delta\xi)}\right]\varphi_{i,j}^{n+1/2};\tag{14}$$

$v>0$，令

$$RHS1=\frac{\varphi_{i,j}^{n}}{\Delta t/2}-\frac{v_{i,j}}{H_2}\frac{\varphi_{i,j}^{n}-\varphi_{i,j-1}^{n}}{\Delta\eta}+R_{i,j}+\frac{\nu z_{i,j+1/2}\varphi_{i,j+1}^{n}-2\nu z_{i,j}\varphi_{i,j}^{n}+\nu z_{i,j-1/2}\varphi_{i,j-1}^{n}}{(H_2)^2(\Delta\eta)^2};\tag{15}$$

$v<0$，令

$$RHS1=\frac{\varphi_{i,j}^{n}}{\Delta t/2}-\frac{v_{i,j}}{H_2}\frac{\varphi_{i,j+1}^{n}-\varphi_{i,j}^{n}}{\Delta\eta}+R_{i,j}+\frac{\nu z_{i,j+1/2}\varphi_{i,j+1}^{n}-2\nu z_{i,j}\varphi_{i,j}^{n}+\nu z_{i,j-1/2}\varphi_{i,j-1}^{n}}{(H_2)^2(\Delta\eta)^2};\tag{16}$$

以上式中，$RHS1$ 和 $RHS2$ 分别代表方程(12)第 1 步离散后的显式右端项和隐式左端项，且 $RHS1=RHS2$.

第 2 步离散步骤与上面类似，具体如下：

$v>0$，令

$$RHS2'=-\frac{\nu z_{i,j+1/2}}{(H_2)^2(\Delta\eta)^2}\varphi_{i,j+1}^{n+1}+\left[\frac{\Delta t}{2}+\frac{2\nu z_{i,j}}{(H_2)^2(\Delta\eta)^2}+\frac{v_{i,j}}{(H_2)(\Delta\eta)}\right]\varphi_{i,j}^{n+1}-\left[\frac{\nu z_{i,j-1/2}}{(H_2)^2(\Delta\eta)^2}+\frac{v_{i,j}}{(H_2)(\Delta\eta)}\right]\varphi_{i,j-1}^{n+1};\tag{17}$$

$v<0$，令

$$RHS2'=-\frac{\nu z_{i,j-1/2}}{(H_2)^2(\Delta\eta)^2}\varphi_{i,j-1}^{n+1}\times\left[-\frac{\nu z_{i,j+1/2}}{(H_2)^2(\Delta\eta)^2}+\frac{v_{i,j}}{(H_2)(\Delta\eta)}\right]\varphi_{i,j+1}^{n+1}+\left[\frac{\Delta t}{2}+\frac{2\nu z_{i,j}}{(H_2)^2(\Delta\eta)^2}-\frac{v_{i,j}}{(H_2)(\Delta\eta)}\right]\varphi_{i,j}^{n+1};\tag{18}$$

$u>0$，令

$$RHS1'=\frac{\varphi_{i,j}^{n}}{\Delta t/2}-\frac{u_{i,j}}{H_1}\frac{\varphi_{i,j}^{n+1/2}-\varphi_{i-1,j}^{n+1/2}}{\Delta\xi}+R_{i,j}+\frac{\nu z_{i+1/2,j}\varphi_{i+1,j}^{n+1/2}-2\nu z_{i,j}\varphi_{i,j}^{n+1/2}+\nu z_{i-1/2,j}\varphi_{i-1,j}^{n+1/2}}{(H_1)^2(\Delta\xi)^2};\tag{19}$$

$u<0$，令

$$RHS1'=\frac{\varphi_{i,j}^{n}}{\Delta t/2}-\frac{u_{i,j}}{H_1}\frac{\varphi_{i+1,j}^{n+1/2}-\varphi_{i,j}^{n+1/2}}{\Delta\xi}+R_{i,j}+\frac{\nu z_{i+1/2,j}\varphi_{i+1,j}^{n+1/2}-2\nu z_{i,j}\varphi_{i,j}^{n+1/2}+\nu z_{i-1/2,j}\varphi_{i-1,j}^{n+1/2}}{(H_1)^2(\Delta\xi)^2};\tag{20}$$

以上式中，$RHS1'$ 和 $RHS2'$ 分别代表方程(12)在 η 向上用隐式差分格式离散和对流项用迎风格式离散显式右端项和隐式左端项，且 $RHS1'=RHS2'$.

上述过程，完成 k 和 ε 计算. 然后，进行第 2 步时间推进，依次逐步推进到下一时刻，反复循环直至计算获得满意结果为止.

3　算例验证

3.1　网格划分

数值计算的模型算例，其物理参数的选取与 De Vriend 的 180°弯道水槽试验模型参数完全一致，即弯道内径为 3.4 m，外径为 5.1 m，出口段和入口段的长度均为 6 m.

整个计算区域的网格数为 101×26，其中 180°弯道的网格数为 61×26，如图 1 所示.

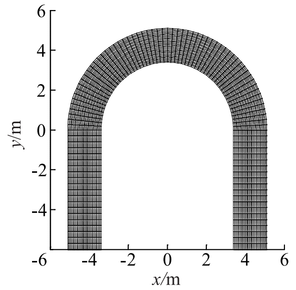

图 1　计算域网格划分分布

Fig. 1　Mesh distributed in flow domain

3.2　计算验证和分析

数值计算的水动力参数也选取与 De Vriend 试验时水动力参数一样，即出流水位为 0.18 m，入流流量为 0.19 m^3/s，且曼宁系数取 $m=0.012$，水流运动黏度为 1.0×10^{-6}.

图 2 为曲线坐标系下二维模型的水位 h 的数值计算结果. 由图可见，由于弯道凹岸水位高于同一截面的凸岸水位，水位进口高、出口低，该物理现象

和 De Vriend 的试验结果基本一致. 图 3 给出了 180°平面弯道模型数值计算结果,即平均速度 u 的等值线分布规律,数值结果与试验结果吻合,且最大绝对误差约为 10^{-3}量级.

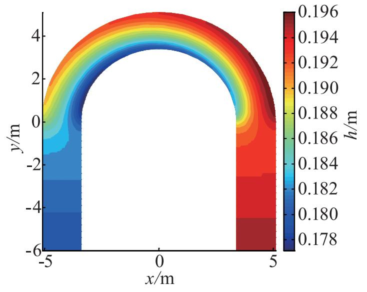

图 2　水位 h 的等值线分布
Fig. 2　Contour of water level

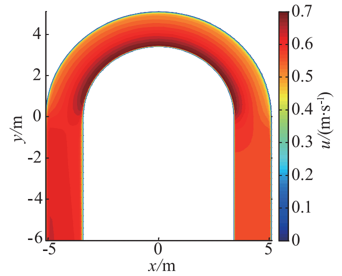

图 3　平均速度 u 的等值线分布
Fig. 3　Contour of mean velocity

图 4 是在不同截面(0°, 30°, 60°, 90°, 120°, 180°)处,数值计算的平均速度 u 随水深 y 的变化规律. 由图可知,计算值与De Vriend的 180° 平面弯道水槽试验结果的实测值吻合较好,且最大绝对误差值约为 10^{-2},因此,数值计算方法是正确、可靠的.

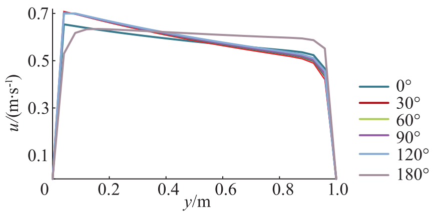

图 4　在不同截面处平均速度 u 随水深演变图
Fig. 4　Evolution of mean velocity profile with depth of water

4　结　论

1) 由于天然河道、海湾等水域边界的复杂性都会遇到在数值计算中边界处理的困惑,为克服该局限推导建立了任意曲线坐标系下的二维浅水方程和 $k-\varepsilon$ 方程.

2) 采用交错有限差分的隐式格式,实现在任意曲线坐标系下的二维浅水方程、$k-\varepsilon$ 方程的数值离散. 经数值计算结果显示,与 De Vriend 的 180°平面弯道水槽试验结果的实测值比对结果吻合较好,这充分说明了该数值方法是正确的,可靠的.

3) 该数值方法适合任意复杂边界的天然水域的水动力计算.

参考文献(References)

[1] Thompson J F, Warsi Z U A, Mastin C W. Boundary-fitted coordinate systems for numerical solution of partial differential equations: A review[J]. Journal of Computational Physics, 1982, 47(1): 1-108.

[2] Stelling G S. On the construction of computational methods for shallow water flow problems[D]. Delft, the Netherlands: Delft University of Technology, 1983.

[3] 王船海,程文辉. 河道二维非恒定流场计算方法研究[J]. 水利学报,1991(1):10-18.
Wang Chuanhai, Cheng Wenhui. The calculation of two dimensional unsteady flow pattern in natural rivers[J]. Journal of Hydraulic Engineering, 1991(1): 10-18. (in Chinese)

[4] De Vriend H J. Mathematical model of steady flow in curved shallow channels[J]. Journal of Hydraulic Research, 1977, 15(1): 37-54.

[5] 王如云, 张东生, 张长宽, 等. 曲线坐标网格下二维涌波数值模拟的 TVD 型格式[J]. 水利学报, 2002(10): 72-77.
Wang Ruyun, Zhang Dongsheng, Zhang Changkuan, et al. Application of TVD scheme to numerical simulation of 2-D surge in curvilinear coordinates[J]. Journal of Hydraulic Engineering, 2002(10): 72-77. (in Chinese)

[6] 黄炳彬,方红卫,刘斌. 复杂边界水流数学模型的斜对角笛卡儿方法[J]. 水动力学研究与进展: A 辑, 2003, 18(6): 679-685.
Huang Bingbin, Fang Hongwei, Liu Bin. Diagonal Cartesian method for numerical simulation of flow with complex boundary[J]. Journal of Hydrodynamics: Ser A, 2003, 18(6): 679-685. (in Chinese)

[7] 吴修广,沈永明,郑永红,等. 非正交曲线坐标下二维水流计算的 SIMPLEC 算法[J]. 水利学报,2003(2): 25-30,37.
Wu Xiuguang, Shen Yongming, Zheng Yonghong, et al. 2-D flow SIMPLEC algorithm in non-orthogonal curvilinear coordinates[J]. Journal of Hydraulic Engineering, 2003(2): 25-30, 37. (in Chinese)

[8] Shi Fengyan, Kirby Js T. Curvilinear parabolic approximation for surface wave transformation with wave-current interaction[J]. Journal of Computational Physics Archive, 2005, 204(2): 562-586.

(责任编辑　张文涛)